# AN APPROACH TO RANKING INTERVAL-VALUED INTUITIONISTIC FUZZY VALUES BASED ON NEW SCORE FUNCTION

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**Abstract:**

**In this paper, we present a new score function *BHF* of interval-valued intuitionistic fuzzy (IVIF) values. The proposed score function *BHF* of IVIF values can overcome the shortcomings of Xu's score function *XUF*, Chen and Chen's score function *DSF*, Chen and Tsai's score function *CF*, Chen and Tsai's score function *KYF* and Yao and Guo's score function *YGF*, which have the drawbacks that they are unable to differentiate the ranking order between IVIF values in several situations.**

**Keywords:**

**IVIF sets; IVIF values; Ranking order; Score function.**

## 1. Introduction

In [1], Atanassov and Gargov presented the theory of interval-valued intuitionistic fuzzy (IVIF) sets through expanding the theory of intuitionistic fuzzy sets (IFSs) [2]. In recent years, some multiattribute decision making methods [3], [4], [5], [6], [7], [9], have been presented based on score functions (SCFNs) of IVIF values. A SCFN of IVIF values can be used to ranking IVIF values. In our daily life, we can rank real values according to their values. The larger the real values, the better the ranking order (RKO) of the real values. In order to ranking IVIF values, we need to transform IVIF values into real values by using a SCFN, and then rank the IVIF values based on the obtained real values, where the larger the obtained real values, the better the RKO of the IVIF values. In [10], Yao and Guo developed a multiattribute group decision making using their proposed SCFN of IVIF values. However, in this paper, we find that the SCFNs of IVIF values presented in [3], [5], [6], [9] and [10] have the drawbacks that they are unable to distinguish the RKO between IVIF values in several situations. Therefore, it is necessary to develop a new SCFN of IVIF values to conquer the drawbacks of the SCFNs of IVIF values presented in [3], [5], [6], [9] and [10] in IVIF environments.

In this paper, we propose a new SCFN of IVIF values. We also show several properties of our developed SCFN of IVIF values. Our developed SCFN of IVIF values is able to conquer the drawbacks of Xu's score function *XUF* [9], Chen and Chen's score function *DSF* [3], Chen and Tsai's score function *CF* [5], Chen and Tsai's score function *KYF* [6] and Yao and Guo's score function *YGF* [10] of IVIF values. The main contribution of this paper is that we propose a novel SCFN of IVIF values, which is able to conquer the shortcomings of the SCFNs of IVIF values presented in [3], [5], [6], [9] and [10].

The rest of this paper is arranged as follows. In Section 2, we present the preliminaries of this paper. In Section 3, we propose a novel SCFN of IVIF values. Furthermore, we also show several properties of our developed SCFN of IVIF values. Section 4 uses some example to show that our proposed SCFN of IVIF values can conquer the shortcomings of the existing SCFNs [3], [5], [6], [9], [10] of IVIF values, which have the drawbacks that they are unable to differentiate RKO between IVIF values in several situations. Section 5 presents the conclusions.

## 2. Preliminaries

In this section, we present the preliminaries of this paper.

***Definition 2.1* [1].** Let $S = \{< y_t, \mu_S(y_t), \upsilon_S(y_t) > | y_t \in Y\}$ be an IVIFS and let $Y = \{y_1, y_2, \dots, y_v\}$ be the universe of discourse, where $\mu_S(y_t)$ and $\upsilon_S(y_t)$ indicate the grades of the membership and the non-membership of $y_t$ in the IVIFS $S$, respectively, $\mu_S(y_t) = [d_t^L, d_t^U]$, $\upsilon_S(y_t) = [g_t^L, g_t^U]$, $0 \le d_t^L \le d_t^U \le 1$, $0 \le g_t^L \le g_t^U \le 1$, $0 \le d_t^U + g_t^U \le 1$ and $t = 1, 2, \ldots, v$.

***Definition 2.2* [9].** Let $S = \{< y_t, \mu_S(y_t), \upsilon_S(y_t) > | y_t \in Y\}$ be an IVIFS in the universe of discourse $Y = \{y_1, y_2, \ldots, y_v\}$, where $\mu_S(y_t) = [d_t^L, d_t^U]$, $\upsilon_S(y_t) = [g_t^L, g_t^U]$ and $t = 1, 2, \ldots, v$. Then, $([d_t^L, d_t^U], [g_t^L, g_t^U])$ is called an IVIF value, where $t = 1, 2, \ldots, v$.

***Definition 2.3* [1], [8].** Let $\tilde{h}_1 = ([d_1^L, d_1^U], [g_1^L, g_1^U])$ and $\tilde{h}_2 = ([d_2^L, d_2^U], [g_2^L, g_2^U])$ be two IVIF values. Then,

(i) If $d_1^L \le d_2^L$, $d_1^U \le d_2^U$, $g_1^L \ge g_2^L$ and $g_1^U \ge g_2^U$, then $\tilde{h}_1 \le \tilde{h}_2$.

(ii) If $d_1^L = d_2^L$, $d_1^U = d_2^U$, $g_1^L = g_2^L$ and $g_1^U = g_2^U$, then $\tilde{h}_1 = \tilde{h}_2$.

***Definition 2.4* [9].** Let $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ be an IVIF value, where $0 \le d^L \le d^U \le 1$, $0 \le g^L \le g^U \le 1$ and $0 \le d^U + g^U \le 1$. Xu's SCFN $XUF$ of the IVIF value $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ is defined as follows:

$$XUF(\tilde{h}) = \frac{d^L - g^L + d^U - g^U}{2}, \quad (1)$$

where $XUF(\tilde{h}) \in [-1, 1]$. A bigger value of $XUF(\tilde{h})$ indicates a bigger IVIF value $\tilde{h}$.

***Definition 2.5* [3].** Let $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ be an IVIF value, where $0 \le d^L \le d^U \le 1$, $0 \le g^L \le g^U \le 1$ and $0 \le d^U + g^U \le 1$. Chen and Chen's SCFN $DSF$ of the IVIF value $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ is presented as follows:

$$DSF(\tilde{h}) = \log(1 + d^L) + \log(1 + d^U) + \log(2 - g^L) + \log(2 - g^U) + sin\left(\frac{\pi}{2} \times \frac{\sqrt{1-g^L} + \sqrt{1-g^U}}{2}\right) + sin\,(\frac{\pi}{2} \times \frac{\sqrt{d^L} + \sqrt{d^U}}{2}), \quad (2)$$

where $DSF(\tilde{h}) \in [0, 6]$. A bigger value of $DSF(\tilde{h})$ indicates a bigger IVIF value $\tilde{h}$.

***Definition 2.6* [5].** Let $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ be an IVIF value, where $0 \le d^L \le d^U \le 1$, $0 \le g^L \le g^U \le 1$ and $0 \le d^U + g^U \le 1$. Chen and Tsai's SCFN $CF$ of the IVIF value $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ is defined as follows:

$$CF(\tilde{h}) = \frac{d^L + d^U - g^L - g^U}{2} + \frac{d^L(1 - g^L) + d^U(1 - g^U)}{2 + g^L + g^U} + 1, \quad (3)$$

where $CF(\tilde{h}) \in [0, 3]$. A bigger value of $CF(\tilde{h})$ indicates a bigger IVIF value $\tilde{h}$.

***Definition 2.7* [6].** Let $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ be an IVIF value, where $0 \le d^L \le d^U \le 1$, $0 \le g^L \le g^U \le 1$ and $0 \le d^U + g^U \le 1$. Chen and Tsai's SCFN $KYF$ of the IVIF value $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ is defined as follows:

$$KYF(\tilde{h}) = \frac{\sqrt{d^L} + \sqrt{d^U} + \sqrt{1 - g^L} + \sqrt{1 - g^U}}{2}, \quad (4)$$

where $KYF(\tilde{h}) \in [0, 2]$. A bigger value of $KYF(\tilde{h})$ indicates a bigger IVIF value $\tilde{h}$.

***Definition 2.8* [10].** Let $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ be an IVIF value, where $0 \le d^L \le d^U \le 1$, $0 \le g^L \le g^U \le 1$ and $0 \le d^U + g^U \le 1$. Yao and Guo's SCFN $YGF$ of the IVIF value $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ is defined as follows:

$$YGF(\tilde{h}) = \frac{1}{6}(2 + d^L + d^U - g^L - g^U + (d^L + d^U)\frac{\sqrt{d^L d^U}}{\sqrt{d^L d^U + g^L g^U}}), \quad (5)$$

where $YGF(\tilde{h}) \in [0, 1]$. A bigger value of $YGF(\tilde{h})$ indicates a bigger IVIF value $\tilde{h}$.

## 3. Our developed score function of IVIF values

In this section, we propose a new score function (SCFN) $BHF$ of interval-valued intuitionistic fuzzy (IVIF) values to conquer the shortcomings of the SCFNs of IVIF values presented in [3], [5], [6], [9] and [10], which have the drawbacks that they are unable to differentiate the ranking order (RKO) between IVIF values in several situations. Our developed SCFN $BHF$ of IVIF values is shown as follows.

***Definition 3.1.*** Let $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ be an IVIF value, where $0 \le d^L \le d^U \le 1$, $0 \le g^L \le g^U \le 1$ and $0 \le d^U + g^U \le 1$. Our developed SCFN $BHF$ of the IVIF value $\tilde{h} = ([d^L, d^U], [g^L, g^U])$ is defined as follows:

$$BHF(\tilde{h}) = \frac{tanh(d^L) + tanh(d^U) + tanh(1 - g^L) + tanh(1 - g^U)}{2} + \frac{ln(d^L + (1 - g^L) + 1) + ln(d^U + (1 - g^U) + 1)}{2}, \quad (6)$$

where $BHF(\tilde{h}) \in [0, 2\,tanh(1) + ln\,(3)]$. A bigger value of $BHF(\tilde{h})$ indicates a bigger IVIF value $\tilde{h}$.

For an interval-valued intuitionistic fuzzy value $\tilde{h} = ([d^L, d^U], [g^L, g^U])$, the bigger the membership value $[d^L, d^U]$, the bigger the score value of the interval-valued intuitionistic fuzzy value $\tilde{h}$; the smaller the non-membership value $[g^L, g^U]$, the bigger the score value of the interval-valued intuitionistic fuzzy value $\tilde{h}$, where $0 \le d^L \le d^U \le 1$, $0 \le g^L \le g^U \le 1$ and $0 \le d^U + g^U \le 1$. Hence, the design principles of the proposed score function $BHF$ of interval-valued intuitionistic fuzzy values presented in Eq. (6) are shown as follows:

(1) The bigger the values of $d^L$ and $d^U$, the bigger the value of $\frac{\tanh(d^L) + \tanh(d^U)}{2}$, resulting in the bigger the value of $BHF(\tilde{h})$.

(2) The lower the values of $g^L$ and $g^U$, the bigger the value of $\frac{\tanh(1 - g^L) + \tanh(1 - g^U)}{2}$, resulting in the bigger the value of $BHF(\tilde{h})$.

(3) The bigger the value of $d^L$ and the lower the value of $g^L$, the bigger the value of $\frac{\ln(d^L + (1 - g^L) + 1)}{2}$, resulting in the bigger the value of $BHF(\tilde{h})$.

(4) The bigger the value of $d^U$ and the lower the value of $g^U$, the bigger the value of $\frac{\ln(d^U+(1-g^U)+1)}{2}$, resulting in the bigger the value of $BHF(\tilde{h})$.

***Property 3.1.*** Let $\tilde{h}_1 = ([d_1^L, d_1^U], [g_1^L, g_1^U])$ and $\tilde{h}_2 = ([d_2^L, d_2^U], [g_2^L, g_2^U])$ be two IVIF values. If $\tilde{h}_1 > \tilde{h}_2$, then $BHF(\tilde{h}_1) > BHF(\tilde{h}_2)$. If $\tilde{h}_1 < \tilde{h}_2$, then $BHF(\tilde{h}_1) < BHF(\tilde{h}_2)$. If $\tilde{h}_1 = \tilde{h}_2$, then $BHF(\tilde{h}_1) = BHF(\tilde{h}_2)$.

**Proof.** Based on Eq. (6), we get

$$BHF(\tilde{h}_1) - BHF(\tilde{h}_2) = \left(\frac{\tanh(d_1^L)+\tanh(d_1^U)+\tanh(1-g_1^L)+\tanh(1-g_1^U)}{2} + \frac{\ln(d_1^L+(1-g_1^L)+1)+\ln(d_1^U+(1-g_1^U)+1)}{2}\right) - \left(\frac{\tanh(d_2^L)+\tanh(d_2^U)+\tanh(1-g_2^L)+\tanh(1-g_2^U)}{2} + \frac{\ln(d_2^L+(1-g_2^L)+1)+\ln(d_2^U+(1-g_2^U)+1)}{2}\right)$$

$$= \left(\frac{\tanh(d_1^L)+\tanh(d_1^U)+\tanh(1-g_1^L)+\tanh(1-g_1^U)}{2} - \frac{\tanh(d_2^L)+\tanh(d_2^U)+\tanh(1-g_2^L)+\tanh(1-g_2^U)}{2}\right) + \left(\frac{\ln(d_1^L+(1-g_1^L)+1)+\ln(d_1^U+(1-g_1^U)+1)}{2} - \frac{\ln(d_2^L+(1-g_2^L)+1)+\ln(d_2^U+(1-g_2^U)+1)}{2}\right)$$

$$= \frac{\tanh(d_1^L)+\tanh(d_1^U) - (\tanh(d_2^L)+\tanh(d_2^U))}{2} + \frac{\tanh(1-g_1^L)+\tanh(1-g_1^U) - (\tanh(1-g_2^L)+\tanh(1-g_2^U))}{2} + \frac{\ln(d_1^L+(1-g_1^L)+1) - \ln(d_2^L+(1-g_2^L)+1)}{2} + \frac{\ln(d_1^U+(1-g_1^U)+1) - \ln(d_2^U+(1-g_2^U)+1)}{2}.$$

(i) If $\tilde{h}_1 > \tilde{h}_2$, where $\tilde{h}_1 = ([d_1^L, d_1^U], [g_1^L, g_1^U])$ and $\tilde{h}_2 = ([d_2^L, d_2^U], [g_2^L, g_2^U])$, then by ***Definition 2.3***, it can be seen that $d_1^L > d_2^L$, $d_1^U > d_2^U$, $g_1^L < g_2^L$, and $g_1^U < g_2^U$. By ***Definition 2.2***, it can be seen that $0 \le d_t^L \le d_t^U \le 1$ and $0 \le g_t^L \le g_t^U \le 1$, where $t = 1, 2$. Then, we get $\frac{\tanh(d_1^L)+\tanh(d_1^U)}{2} > \frac{\tanh(d_2^L)+\tanh(d_2^U)}{2}$, $\frac{\tanh(1-g_1^L)+\tanh(1-g_1^U)}{2} > \frac{\tanh(1-g_2^L)+\tanh(1-g_2^U)}{2}$, $\frac{\ln(d_1^L+(1-g_1^L)+1)}{2} > \frac{\ln(d_2^L+(1-g_2^L)+1)}{2}$, and $\frac{\ln(d_1^U+(1-g_1^U)+1)}{2} > \frac{\ln(d_2^U+(1-g_2^U)+1)}{2}$. Therefore, $\frac{\tanh(d_1^L)+\tanh(d_1^U)}{2} - \frac{\tanh(d_2^L)+\tanh(d_2^U)}{2} > 0$, $\frac{\tanh(1-g_1^L)+\tanh(1-g_1^U)}{2} - \frac{\tanh(1-g_2^L)+\tanh(1-g_2^U)}{2} > 0$, $\frac{\ln(d_1^L+(1-g_1^L)+1)}{2} - \frac{\ln(d_2^L+(1-g_2^L)+1)}{2} > 0$, and $\frac{\ln(d_1^U+(1-g_1^U)+1)}{2} - \frac{\ln(d_2^U+(1-g_2^U)+1)}{2} > 0$. Because $BHF(\tilde{h}_1) - BHF(\tilde{h}_2) = \frac{\tanh(d_1^L)+\tanh(d_1^U) - (\tanh(d_2^L)+\tanh(d_2^U))}{2} + \frac{\tanh(1-g_1^L)+\tanh(1-g_1^U) - (\tanh(1-g_2^L)+\tanh(1-g_2^U))}{2} + \frac{\ln(d_1^L+(1-g_1^L)+1) - \ln(d_2^L+(1-g_2^L)+1)}{2} + \frac{\ln(d_1^U+(1-g_1^U)+1) - \ln(d_2^U+(1-g_2^U)+1)}{2}$, because $\frac{\tanh(d_1^L)+\tanh(d_1^U)}{2} - \frac{\tanh(d_2^L)+\tanh(d_2^U)}{2} > 0$, because $\frac{\tanh(1-g_1^L)+\tanh(1-g_1^U)}{2} - \frac{\tanh(1-g_2^L)+\tanh(1-g_2^U)}{2} > 0$, because $\frac{\ln(d_1^L+(1-g_1^L)+1)}{2} - \frac{\ln(d_2^L+(1-g_2^L)+1)}{2} > 0$, and because $\frac{\ln(d_1^U+(1-g_1^U)+1)}{2} - \frac{\ln(d_2^U+(1-g_2^U)+1)}{2} > 0$, we get $BHF(\tilde{h}_1) - BHF(\tilde{h}_2) > 0$. That is, $BHF(\tilde{h}_1) > BHF(\tilde{h}_2)$. Therefore, if the IVIF values $\tilde{h}_1 > \tilde{h}_2$, then $BHF(\tilde{h}_1) > BHF(\tilde{h}_2)$.

(ii) If $\tilde{h}_1 = \tilde{h}_2$, where $\tilde{h}_1 = ([d_1^L, d_1^U], [g_1^L, g_1^U])$ and $\tilde{h}_2 = ([d_2^L, d_2^U], [g_2^L, g_2^U])$, then by ***Definition 2.3***, it can be seen that $d_1^L = d_2^L$, $d_1^U = d_2^U$, $g_1^L = g_2^L$, and $g_1^U = g_2^U$. Then, we get $\frac{\tanh(d_1^L)+\tanh(d_1^U)}{2} = \frac{\tanh(d_2^L)+\tanh(d_2^U)}{2}$, $\frac{\tanh(1-g_1^L)+\tanh(1-g_1^U)}{2} = \frac{\tanh(1-g_2^L)+\tanh(1-g_2^U)}{2}$, $\frac{\ln(d_1^L+(1-g_1^L)+1)}{2} = \frac{\ln(d_2^L+(1-g_2^L)+1)}{2}$, and $\frac{\ln(d_1^U+(1-g_1^U)+1)}{2} = \frac{\ln(d_2^U+(1-g_2^U)+1)}{2}$. Therefore, $\frac{\tanh(d_1^L)+\tanh(d_1^U)}{2} - \frac{\tanh(d_2^L)+\tanh(d_2^U)}{2} = 0$, $\frac{\tanh(1-g_1^L)+\tanh(1-g_1^U)}{2} - \frac{\tanh(1-g_2^L)+\tanh(1-g_2^U)}{2} = 0$, $\frac{\ln(d_1^L+(1-g_1^L)+1)}{2} - \frac{\ln(d_2^L+(1-g_2^L)+1)}{2} = 0$, and $\frac{\ln(d_1^U+(1-g_1^U)+1)}{2} - \frac{\ln(d_2^U+(1-g_2^U)+1)}{2} = 0$. Because $BHF(\tilde{h}_1) - BHF(\tilde{h}_2) = \frac{\tanh(d_1^L)+\tanh(d_1^U) - (\tanh(d_2^L)+\tanh(d_2^U))}{2} + \frac{\tanh(1-g_1^L)+\tanh(1-g_1^U) - (\tanh(1-g_2^L)+\tanh(1-g_2^U))}{2} + \frac{\ln(d_1^L+(1-g_1^L)+1) - \ln(d_2^L+(1-g_2^L)+1)}{2} + \frac{\ln(d_1^U+(1-g_1^U)+1) - \ln(d_2^U+(1-g_2^U)+1)}{2}$, because $\frac{\tanh(d_1^L)+\tanh(d_1^U)}{2} - \frac{\tanh(d_2^L)+\tanh(d_2^U)}{2} = 0$, because $\frac{\tanh(1-g_1^L)+\tanh(1-g_1^U)}{2} - \frac{\tanh(1-g_2^L)+\tanh(1-g_2^U)}{2} = 0$, because $\frac{\ln(d_1^L+(1-g_1^L)+1)}{2} - \frac{\ln(d_2^L+(1-g_2^L)+1)}{2} = 0$, and because $\frac{\ln(d_1^U+(1-g_1^U)+1)}{2} - \frac{\ln(d_2^U+(1-g_2^U)+1)}{2} = 0$, we get $BHF(\tilde{h}_1) - BHF(\tilde{h}_2) = 0$. That is, $BHF(\tilde{h}_1) = BHF(\tilde{h}_2)$. Therefore, if the IVIF values $\tilde{h}_1 = \tilde{h}_2$, then $BHF(\tilde{h}_1) = BHF(\tilde{h}_2)$.

***Property 3.2.*** Let $\tilde{h}$ be an IVIF value, where $\tilde{h} = ([d^L, d^U], [g^L, g^U])$, $0 \le d^L \le d^U \le 1$, $0 \le g^L \le g^U \le 1$, and $0 \le d^U + g^U \le 1$. Then, $BHF(\tilde{h}) \in [0, 2\tanh(1) + ln\,(3)]$.

***Property 3.3.*** If the IVIF value $\tilde{h} = ([0,0],[1,1])$, then $BHF(\tilde{h}) = 0$.

***Property 3.4.*** If the IVIF value $\tilde{h} = ([1,1],[0,0])$, then $BHF(\tilde{h}) = 2\tanh(1) + ln\,(3)$.

## 4. Examples

***Example 4.1***: Let $\tilde{h}_1 = ([0, 0.10], [0, 0.55])$ and $\tilde{h}_2 = ([0, 0.15], [0.20, 0.50])$ be two IVIF values. Then,

(1) By using the SCFN $XUF$ [9] presented in Eq. (1), it obtains $XUF(\tilde{h}_1) = -0.225$ and $XUF(\tilde{h}_2) = -0.275$. Since $XUF(\tilde{h}_1) > XUF(\tilde{h}_2)$, where $XUF(\tilde{h}_1) = -0.225$ and $XUF(\tilde{h}_2) = -0.275$, the SCFN $XUF$ [9] obtains the RKO "$\tilde{h}_1 > \tilde{h}_2$" for the IVIF values $\tilde{h}_1$ and $\tilde{h}_2$.

(2) By using the SCFN $DSF$ [3] presented in Eq. (2), it obtains $DSF(\tilde{h}_1) = 2.886$ and $DSF(\tilde{h}_2) = 2.886$. Since $DSF(\tilde{h}_1) = DSF(\tilde{h}_2)$, where $DSF(\tilde{h}_1) = 2.886$ and $DSF(\tilde{h}_2) = 2.886$, the SCFN $DSF$ [3] obtains the RKO "$\tilde{h}_1 = \tilde{h}_2$" for the IVIF values $\tilde{h}_1$ and $\tilde{h}_2$. Hence, the SCFN $DSF$ [3] has the shortcoming that it is unable to differentiate the RKO between the IVIF values $\tilde{h}_1$ and $\tilde{h}_2$ in this circumstance.

(3) By using the SCFN $CF$ [5] presented in Eq. (3), it obtains $CF(\tilde{h}_1) = 0.793$ and $CF(\tilde{h}_2) = 0.753$. Since $CF(\tilde{h}_1) > CF(\tilde{h}_2)$, where $CF(\tilde{h}_1) = 0.793$ and $CF(\tilde{h}_2) = 0.753$, the SCFN $CF$ [5] obtains the RKO "$\tilde{h}_1 > \tilde{h}_2$" for the IVIF values $\tilde{h}_1$ and $\tilde{h}_2$.

(4) By using the SCFN $KYF$ [6] presented in Eq. (4), it obtains $KYF(\tilde{h}_1) = 0.994$ and $KYF(\tilde{h}_2) = 0.994$. Since $KYF(\tilde{h}_1) = KYF(\tilde{h}_2)$, where $KYF(\tilde{h}_1) = 0.994$ and $KYF(\tilde{h}_2) = 0.994$, the SCFN $KYF$ [6] obtains the RKO "$\tilde{h}_1 = \tilde{h}_2$" for the IVIF values $\tilde{h}_1$ and $\tilde{h}_2$. Hence, the SCFN $KYF$ [3] has the shortcoming that it is unable to differentiate the RKO between the IVIF values $\tilde{h}_1$ and $\tilde{h}_2$ in this circumstance.

(5) By using the SCFN $YGF$ [10] presented in Eq. (5), it obtains $YGF(\tilde{h}_1) = 0.261$ and $YGF(\tilde{h}_2) = 0.242$. Since $YGF(\tilde{h}_1) > YGF(\tilde{h}_2)$, where $YGF(\tilde{h}_1) = 0.261$ and $YGF(\tilde{h}_2) = 0.242$, the SCFN $YGF$ [10] obtains the RKO "$\tilde{h}_1 > \tilde{h}_2$" for the IVIF values $\tilde{h}_1$ and $\tilde{h}_2$.

(6) By using the proposed SCFN $BHF$ presented in Eq. (6), we obtain $BHF(\tilde{h}_1) = 1.207$ and $BHF(\tilde{h}_2) = 1.182$. Since $BHF(\tilde{h}_1) > BHF(\tilde{h}_2)$, where $BHF(\tilde{h}_1) = 1.207$ and $BHF(\tilde{h}_2) = 1.182$, the proposed SCFN $BHF$ obtains the RKO "$\tilde{h}_1 > \tilde{h}_2$" for the IVIF values $\tilde{h}_1$ and $\tilde{h}_2$.

***Example 4.2***: Let $\tilde{h}_3 = ([0.60, 0.81], [0.01, 0.15])$ and $\tilde{h}_4 = ([0.69, 0.81], [0.11, 0.13])$ be two IVIF values. Then,

(1) By using the SCFN $XUF$ [9] presented in Eq. (1), it obtains $XUF(\tilde{h}_3) = 0.625$ and $XUF(\tilde{h}_4) = 0.630$. Since $XUF(\tilde{h}_3) < XUF(\tilde{h}_4)$, where $XUF(\tilde{h}_3) = 0.625$ and $XUF(\tilde{h}_4) = 0.630$, the SCFN $XUF$ [9] obtains the RKO "$\tilde{h}_3 < \tilde{h}_4$" for the IVIF values $\tilde{h}_3$ and $\tilde{h}_4$.

(2) By using the SCFN $DSF$ [3] presented in Eq. (2), it obtains $DSF(\tilde{h}_3) = 5.380$ and $DSF(\tilde{h}_4) = 5.407$. Since $DSF(\tilde{h}_3) < DSF(\tilde{h}_4)$, where $DSF(\tilde{h}_3) = 5.380$ and $DSF(\tilde{h}_4) = 5.407$, the SCFN $DSF$ [3] obtains the RKO "$\tilde{h}_3 < \tilde{h}_4$" for the IVIF values $\tilde{h}_3$ and $\tilde{h}_4$.

(3) By using the SCFN $CF$ [5] presented in Eq. (3), it obtains $CF(\tilde{h}_3) = 2.219$ and $CF(\tilde{h}_4) = 2.219$. Since $CF(\tilde{h}_3) = CF(\tilde{h}_4)$, where $CF(\tilde{h}_3) = 2.219$ and $CF(\tilde{h}_4) = 2.219$, the SCFN $CF$ [5] obtains the RKO "$\tilde{h}_3 = \tilde{h}_4$" for the IVIF values $\tilde{h}_3$ and $\tilde{h}_4$. Hence, the SCFN $CF$ [5] has the shortcoming that it is unable to differentiate the RKO between the IVIF values $\tilde{h}_3$ and $\tilde{h}_4$ in this circumstance.

(4) By using the SCFN $KYF$ [6] presented in Eq. (4), it obtains $KYF(\tilde{h}_3) = 1.796$ and $KYF(\tilde{h}_4) = 1.803$. Since $KYF(\tilde{h}_3) < KYF(\tilde{h}_4)$, where $KYF(\tilde{h}_3) = 1.796$ and $KYF(\tilde{h}_4) = 1.803$, the SCFN $KYF$ [6] obtains the RKO "$\tilde{h}_3 < \tilde{h}_4$" for the IVIF fuzzy values $\tilde{h}_3$ and $\tilde{h}_4$.

(5) By using the SCFN $YGF$ [10] presented in Eq. (5), it obtains $YGF(\tilde{h}_3) = 0.776$ and $YGF(\tilde{h}_4) = 0.790$. Since $YGF(\tilde{h}_3) < YGF(\tilde{h}_4)$, where $YGF(\tilde{h}_3) = 0.776$ and $YGF(\tilde{h}_4) = 0.790$, the SCFN $YGF$ [10] obtains the RKO "$\tilde{h}_3 < \tilde{h}_4$" for the IVIF values $\tilde{h}_3$ and $\tilde{h}_4$.

(6) By using the proposed SCFN $BHF$ presented in Eq. (6), we obtain $BHF(\tilde{h}_3) = 2.293$ and $(\tilde{h}_4) = 2.307$. Since $BHF(\tilde{h}_3) < BHF(\tilde{h}_4)$, where $BHF(\tilde{h}_3) = 2.293$ and $BHF(\tilde{h}_4) = 2.307$, the proposed SCFN $BHF$ obtains the RKO "$\tilde{h}_3 < \tilde{h}_4$" for the IVIF values $\tilde{h}_3$ and $\tilde{h}_4$.

***Example 4.3***: Let $\tilde{h}_5 = ([0.02, 0.02], [0.05, 0.34])$ and $\tilde{h}_6 = ([0, 0.08], [0.05, 0.34])$ be two IVIF values. Then,

(1) By using the SCFN $XUF$ [9] presented in Eq. (1), it obtains $XUF(\tilde{h}_5) = -0.175$ and $XUF(\tilde{h}_6) = -0.155$. Since $XUF(\tilde{h}_5) < XUF(\tilde{h}_6)$, the SCFN $XUF$ [9] obtains the RKO "$\tilde{h}_5 < \tilde{h}_6$" for the IVIF values $\tilde{h}_5$ and $\tilde{h}_6$.

(2) By using the SCFN $DSF$ [3] presented in Eq. (2), it obtains $DSF(\tilde{h}_5) = 2.958$ and $DSF(\tilde{h}_6) = 3.012$. Since $DSF(\tilde{h}_5) < DSF(\tilde{h}_6)$, the SCFN $DSF$ [3] obtains

the RKO "$\tilde{h}_5 < \tilde{h}_6$" for the IVIF values $\tilde{h}_5$ and $\tilde{h}_6$.

(3) By using the SCFN $CF$ [5] presented in Eq. (3), it obtains $CF(\tilde{h}_5) =$ and $CF(\tilde{h}_6) = 0.867$. Since $CF(\tilde{h}_5) < CF(\tilde{h}_6)$, the SCFN $CF$ [5] obtains the RKO "$\tilde{h}_5 < \tilde{h}_6$" for the IVIF values $\tilde{h}_5$ and $\tilde{h}_6$.

(4) By using the SCFN $KYF$ [6] presented in Eq. (4), it obtains $KYF(\tilde{h}_5) = 1.035$ and $KYF(\tilde{h}_6) = 1.035$. Since $KYF(\tilde{h}_5) = KYF(\tilde{h}_6)$, where $KYF(\tilde{h}_5) = 1.035$ and $KYF(\tilde{h}_6) = 1.035$, the SCFN $KYF$ [6] obtains the RKO "$\tilde{h}_5 = \tilde{h}_6$" for the IVIF values $\tilde{h}_5$ and $\tilde{h}_6$. Hence, the SCFN $KYF$ [6] has the shortcoming that it is unable to differentiate the RKO between the IVIF values $\tilde{h}_5$ and $\tilde{h}_6$ in this circumstance.

(5) By using the SCFN $YGF$ [10] presented in Eq. (5), it obtains $YGF(\tilde{h}_5) = 0.276$ and $YGF(\tilde{h}_6) = 0.282$. Since $YGF(\tilde{h}_5) < YGF(\tilde{h}_6)$, where $YGF(\tilde{h}_5) = 0.276$ and $YGF(\tilde{h}_6) = 0.282$, the SCFN $YGF$ [10] obtains the RKO "$\tilde{h}_5 < \tilde{h}_6$" for the IVIF values $\tilde{h}_5$ and $\tilde{h}_6$.

(6) By using the proposed SCFN $BHF$ presented in Eq. (6), we obtain $BHF(\tilde{h}_5) = 1.277$ and $BHF(\tilde{h}_6) = 1.310$. Since $BHF(\tilde{h}_5) < BHF(\tilde{h}_6)$, where $BHF(\tilde{h}_5) = 1.277$ and $BHF(\tilde{h}_6) = 1.310$, the proposed SCFN $BHF$ obtains the RKO "$\tilde{h}_5 < \tilde{h}_6$" for the IVIF values $\tilde{h}_5$ and $\tilde{h}_6$.

***Example 4.4***: Let $\tilde{h}_7 = ([0, 0.42], [0.38, 0.54])$ and $\tilde{h}_8 = ([0, 0.14], [0.23, 0.41])$ be two IVIF values. Then,

(1) By using the SCFN $XUF$ [9] presented in Eq. (1), it obtains $XUF(\tilde{h}_7) = -0.250$ and $XUF(\tilde{h}_8) = -0.250$. Since $XUF(\tilde{h}_7) = XUF(\tilde{h}_8)$, where $XUF(\tilde{h}_7) = -0.250$ and $XUF(\tilde{h}_8) = -0.250$, the SCFN $XUF$ [9] obtains the RKO "$\tilde{h}_7 = \tilde{h}_8$" for the IVIF values $\tilde{h}_7$ and $\tilde{h}_8$. Hence, the SCFN $XUF$ [9] has the shortcoming that is unable to differentiate the RKO between the IVIF values $\tilde{h}_7$ and $\tilde{h}_8$ in this circumstance.

(2) By using the SCFN $DSF$ [3] presented in Eq. (2), it obtains $DSF(\tilde{h}_7) = 3.148$ and $DSF(\tilde{h}_8) = 2.933$. Since $DSF(\tilde{h}_7) > DSF(\tilde{h}_8)$, where $DSF(\tilde{h}_7) = 3.148$ and $DSF(\tilde{h}_8) = 2.933$, the SCFN $DSF$ [3] obtains the RKO "$\tilde{h}_7 > \tilde{h}_8$" for the IVIF values $\tilde{h}_7$ and $\tilde{h}_8$.

(3) By using the SCFN $CF$ [5] presented in Eq. (3), it obtains $CF(\tilde{h}_7) = 0.816$ and $CF(\tilde{h}_8) = 0.781$. Since $CF(\tilde{h}_7) > CF(\tilde{h}_8)$, where $CF(\tilde{h}_7) = 0.816$ and $CF(\tilde{h}_8) = 0.781$, the SCFN $CF$ [5] acquires the RKO "$\tilde{h}_7 > \tilde{h}_8$" for the IVIF values $\tilde{h}_7$ and $\tilde{h}_8$.

(4) By using the SCFN $KYF$ [6] presented in Eq. (4), it obtains $KYF(\tilde{h}_7) = 1.057$ and $KYF(\tilde{h}_8) = 1.010$. Since $KYF(\tilde{h}_7) > KYF(\tilde{h}_8)$, where $KYF(\tilde{h}_7) = 1.057$ and $KYF(\tilde{h}_8) = 1.010$, the SCFN $KYF$ [6] obtains the RKO "$\tilde{h}_7 > \tilde{h}_8$" for the IVIF values $\tilde{h}_7$ and $\tilde{h}_8$.

(5) By using the SCFN $YGF$ [10] presented in Eq. (5), it obtains $YGF(\tilde{h}_7) = 0.250$ and $YGF(\tilde{h}_8) = 0.250$. Since $YGF(\tilde{h}_7) = YGF(\tilde{h}_8)$, where $YGF(\tilde{h}_7) = 0.250$ and $YGF(\tilde{h}_8) = 0.250$, the SCFN $YGF$ [10] obtains the RKO "$\tilde{h}_7 = \tilde{h}_8$" for the IVIF values $\tilde{h}_7$ and $\tilde{h}_8$. Hence, the SCFN $YGF$ [10] has the shortcoming that it is unable to differentiate the RKO between the IVIF values $\tilde{h}_7$ and $\tilde{h}_8$ in this circumstance.

(6) By using the proposed SCFN $BHF$ presented in Eq. (6), we obtain $BHF(\tilde{h}_7) = 1.246$ and $BHF(\tilde{h}_8) = 1.218$. Since $BHF(\tilde{h}_7) > BHF(\tilde{h}_8)$, where $BHF(\tilde{h}_7) = 1.246$ and $BHF(\tilde{h}_8) = 1.218$, the proposed SCFN $BHF$ obtains the RKO "$\tilde{h}_7 > \tilde{h}_8$" for the IVIF values 7 and $\tilde{h}_8$.

Table 1 shows a comparison of the RKOs of IVIF values obtained by various SCFNs of IVIF values. From Table 1, we can observe that our developed SCFN $BHF$ of IVIF values can overcome the drawbacks of the SCFN $XUF$ [9], the SCFN $DSF$ [3], the SCFN $CF$ [5], the SCFN $KYF$ [6] and the SCFN $YGF$ [10] of IVIF values.

**TABLE 1.** A comparison of the RKOs of the IVIF values obtained by various SCFNs

| SCFNs / IVIF Values | $XUF$ [9] | $DSF$ [3] | $CF$ [5] | $KYF$ [6] | $YGF$ [10] | Our Developed SCFN $BHF$ |
|---|---|---|---|---|---|---|
| $\tilde{h}_1 = ([0, 0.1], [0, 0.55])$ <br> $\tilde{h}_2 = ([0, 0.15], [0.2, 0.5])$ | $\tilde{h}_1 > \tilde{h}_2$ | $\tilde{h}_1 = \tilde{h}_2$ | $\tilde{h}_1 > \tilde{h}_2$ | $\tilde{h}_1 = \tilde{h}_2$ | $\tilde{h}_1 > \tilde{h}_2$ | $\tilde{h}_1 > \tilde{h}_2$ |
| $\tilde{h}_3 = ([0.6, 0.81], [0.01, 0.15])$ <br> $\tilde{h}_4 = ([0.69, 0.81], [0.11, 0.13])$ | $\tilde{h}_3 < \tilde{h}_4$ | $\tilde{h}_3 < \tilde{h}_4$ | $\tilde{h}_3 = \tilde{h}_4$ | $\tilde{h}_3 < \tilde{h}_4$ | $\tilde{h}_3 < \tilde{h}_4$ | $\tilde{h}_3 < \tilde{h}_4$ |
| $\tilde{h}_5 = ([0.02, 0.02], [0.05, 0.34])$ <br> $\tilde{h}_6 = ([0, 0.08], [0.05, 0.34])$ | $\tilde{h}_5 < \tilde{h}_6$ | $\tilde{h}_5 < \tilde{h}_6$ | $\tilde{h}_5 < \tilde{h}_6$ | $\tilde{h}_5 = \tilde{h}_6$ | $\tilde{h}_5 < \tilde{h}_6$ | $\tilde{h}_5 < \tilde{h}_6$ |
| $\tilde{h}_7 = ([0, 0.42], [0.38, 0.54])$ <br> $\tilde{h}_8 = ([0, 0.14], [0.23, 0.41])$ | $\tilde{h}_7 = \tilde{h}_8$ | $\tilde{h}_7 > \tilde{h}_8$ | $\tilde{h}_7 > \tilde{h}_8$ | $\tilde{h}_7 > \tilde{h}_8$ | $\tilde{h}_7 = \tilde{h}_8$ | $\tilde{h}_7 > \tilde{h}_8$ |

Note: ▢ denotes the RKO between IVIF values cannot be differentiated.

## 5. Conclusions

In this paper, we have developed a new score function (SCFN) $BHF$ of interval-valued intuitionistic fuzzy (IVIF) values. The proposed SCFN $BHF$ of IVIF values can overcome the shortcomings of Xu's SCFN $XUF$ [9], Chen and Chen's SCFN $DSF$ [3], Chen and Tsai's SCFN $CF$ [5], Chen and Tsai's SCFN $KYF$ [6] and Yao and Guo's SCFN $YGF$ [10] of IVIF values, which have the drawbacks that they are unable to differentiate the ranking order between IVIF values in several situations.

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